A TIME VARYING STRONG COUPLING CONSTANT AS A MODEL OF INFLATIONARY UNIVERSE

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Abstract

We consider a scenario where the strong coupling constant was changing in the early universe. We attribute this change to a variation in the colour charge within a Bekenstein-like model. Treating the vacuum gluon condensate $< G^2 >$ as a free paramete r , we could generate inflation with the required properties to solve the fluctuation and other standard cosmology problems. A possible approach to end the inflation is suggested.

Key words: fundamental constants; inflationary models; cosmology; QCD

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1 Introduction

The hypothesis that the universe underwent a period of exponential expansion at very early times is the most popular theory of the early universe. The "old" [1] and "new" [2, 3] inflationary universe models were able to solve the "horizon", "flatness" and "structure formation" problems creating, in turn, their own problems, some of which the "chaotic" model [4] and its extensions [5] could tackle. These models are all based on the use of new fundamental scalar fields, the "inflatons", which can not be the Higgs fields of ordinary gauge theories. Later, other possible alternatives t o inflationary cosmology were proposed assuming, rather than changing the matter content of the universe, a change in the speed of light [6, 7] or, more generally, a varying fine structure constant α_{em} [8]. In a recent work [9], we have generalized Bekenstein model [10] for the time variation of α_{em} to QCD strong coupling constant α_S and found that experimental constraints going backward till quasar formation times rule out α_S variability. In this letter, we discuss how our "minimal" Bekenstein-like model for α_S, when implemented in the very early universe, can provide a realization of inflation driven by the trace anomaly of QCD energy momentum tensor. Inflationary models driven by the trace anomaly of conformally coupled matter fields are treated in the literature [11] while in our model we concentrate on the gauge fields of QCD. Assuming the universe is radiation-dominat ed at early times and that vacuum expectation values predominate over matter densities, we find, with suitable values of the gluon condensate $< G^2 >$ far larger than its present value and the Bekenstein length scale l far smaller than the Planck-Wheeler length scale L_P, that our inflation is self-consistent with acceptable numerical results to solve the fluctuation and other problems. We shall not dwell on the possible mechanisms by which the gluon condensate could have decreased to reach its present value. Rather we wish to concentrate on the conditions we should impose on our model to be interesting. We hope this phenomenological approach could prompt further work on a possible connection between time-varying fundamental "constants" and inflationary models.

2 Analysis

In [9]we used the QCD Lagrangian with a varying coupling "constant"

$$\begin{aligned} L_{QCD} &= L_\epsilon + L_g + L_m \\ &= -\frac{1}{2l^2}\frac{\epsilon_{,\mu}\epsilon^{,\mu}}{\epsilon^2} - \frac{1}{2}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_f \bar{\psi}^{(f)}(i\gamma^\mu D_\mu - M_f)\psi^{(f)} \end{aligned} \tag{1}$$

where l is the Bekenstein scale length, $\epsilon(x)$ is a scalar gauge-invariant and dimensionless field representing the variation of the strong coupling "constant" $g(x) = g_0\epsilon(x)$, $D_\mu = \partial_\mu - ig_0\epsilon(x)A_\mu$ is the covariant derivative and where the gluon tensor field should be given by

$$G^a_{\mu\nu} = \frac{1}{\epsilon}[\partial_\mu(\epsilon A^a_\nu) - \partial_\nu(\epsilon A^a_\mu) + g_0\epsilon^2 f^{abc}A^b_\mu A^c_\nu] \tag{2}$$

We assume homogeneity and isotropy for an expanding universe and so consider only temporal variations for α_S. We assume also, rather plausibly in the radiation-dominated early universe, negligence of matter contribution to get the following equations of motion

$$(\frac{G^{\mu\nu}_a}{\epsilon})_{;\mu} - g_0 f^{abc}G^{\mu\nu}_b A^c_\mu + \sum_f g_0\bar{\psi}t^a\gamma^\nu\psi = 0 \tag{3}$$

$$(a^3\frac{\dot{\epsilon}}{\epsilon})^{\cdot} = \frac{a^3(t)l^2}{2}\langle G^2\rangle \tag{4}$$

where $a(t)$ is the expansion scale factor in R-W metric.

Computing the canonical energy-momentum tensor $\frac{\partial L}{\partial(\partial_\alpha\phi_i)}\partial^\beta\phi_i - g^{\alpha\beta}L$ we get a non gauge invariant quantity. This may be cured by a standard technique [12] amounting to a subtraction of a total derivative and hence not changing the equations of motion. We subtract the total derivative

$$\Delta T^{\alpha\beta} = \partial_\nu\left(-\frac{G^{\alpha\nu}_a}{\epsilon}\epsilon A^{a\beta}\right)$$

to get, with the use of equation (3), the gauge-invariant energy momentum tensor

$$\begin{aligned} T^{\alpha\beta} &= G^{\alpha\nu}_a G^{a\beta}_\nu + i\sum_f \bar{\psi}^{(f)}\gamma^{(\alpha}D^{\beta)}\psi^{(f)} - \frac{1}{l^2}\frac{\partial^\alpha\epsilon\partial^\beta\epsilon}{\epsilon^2} \\ &\quad - g^{\alpha\beta}\left[-\frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu} + \sum_f \bar{\psi}^{(f)}(i\gamma^\mu D_\mu - M_f)\psi^{(f)} - \frac{1}{2l^2}\frac{\epsilon_{,\mu}\epsilon^{,\mu}}{\epsilon^2}\right] \end{aligned} \tag{5}$$

Since we assume radiation dominated era during the very early universe we can concentrate on the gauge fields and neglect the matter fields contribution and so we decompose our energy momentum tensor into two parts

$$
\begin{aligned}
T_{\alpha\beta} &= T^{QCD}_{\alpha\beta} + T^{\epsilon}_{\alpha\beta} \qquad (6)\\
&= \left(G_{\alpha\nu}G^{\nu}_{\beta} - g_{\alpha\beta}\left[-\frac{1}{4}G^{\mu\nu}_{a}G^{a}_{\mu\nu}\right]\right) + \left(-\frac{1}{l^2}\frac{\partial_\alpha\epsilon\partial_\beta\epsilon}{\epsilon^2} - g_{\alpha\beta}\left[-\frac{1}{2l^2}\frac{\epsilon_{,\mu}\epsilon^{,\mu}}{\epsilon^2}\right]\right)
\end{aligned}
$$

The $T^{\epsilon}_{\alpha\beta}$ would lead to an energy density $T^{\epsilon}_{00} = \rho_\epsilon = -\frac{1}{2l^2}(\frac{\dot{\epsilon}}{\epsilon})^2$ and a pressure $T^{\epsilon}_{ij} = g^{(3)}_{ij}p_\epsilon$ of equal value, while for the $T^{QCD}_{\alpha\beta}$, though containing ϵ via the expression of G, we shall treat it like a pure ordinary QCD. More precisely, when decomposing the energy-momentum tensor $T^{QCD}_{\alpha\beta}$ into trace and traceless parts, we can write the corresponding energy mass density $\rho_{QCD} = T^{QCD}_{00}$ and the pressure p_{QCD}, like in "ordinary" QCD, as a sum of two terms

$$
\begin{aligned}
\rho_{QCD} &= \rho^{r}_{QCD} + \rho^{T}_{QCD} \qquad (7)\\
p_{QCD} &= p^{r}_{QCD} + p^{T}_{QCD} \qquad (8)
\end{aligned}
$$

where ρ^{r}_{QCD} is the density corresponding to the "traceless" part of the gauge field satisfying

$$\rho^{r}_{QCD} = 3p^{r}_{QCD} \qquad (9)$$

while the trace part of the gauge field energy-momentum tensor, being proportional to $g_{\alpha\beta}$ and hence behaving like a 'cosmological constant' term, would give a mass density ρ^{T}_{QCD} and a pressure p^{T}_{QCD} of opposite values

$$\rho^{T}_{QCD} = -p^{T}_{QCD} \qquad (10)$$

We shall assume, here, that the trace anomaly relation for $T^{\mu QCD}_{\mu}$ is identical in form to the "ordinary" QCD trace anomaly which is given, up to leading order in the time-varying coupling "constant" $\alpha_S = \alpha_{S_0}\epsilon^2$, by [13]

$$T^{\mu QCD}_{\mu} = \rho_{QCD} - 3p_{QCD} = -\frac{9\alpha_{S_0}\epsilon^2}{8\pi}G^{\mu\nu}_{a}G^{a}_{\mu\nu} \qquad (11)$$

Again, neglecting matter contribution during the radiation dominated era, we limit our gluon operator $G^{\mu\nu}_{a}G^{a}_{\mu\nu}$ matrix elements to its condensate vacuum expectation value $< G^2 >$.

Thus, we can write the total energy mass density as

$$\rho = \rho^r_{QCD} + \rho^T_{QCD} + \rho_\epsilon \tag{12}$$

and equation (10) would suggest that in our model, in analogy to ordinary inflationary models, the QCD trace anomaly would generate the inflation during which the total energy mass density stays approximately constant. For this, let us suppose the "trace-anomaly" energy mass density dominating over the other densities

$$\rho^T_{QCD} \quad >> \quad \rho_\epsilon, \rho^r_{QCD} \tag{13}$$

and we should seek a consistent inflationary solution ($a(t) \sim e^{Ht}$ and $H \equiv \frac{\dot{a}}{a}$ is approximately constant) to the FRW equations in a flat space-time (with $k = 0$)

$$\begin{aligned} (\frac{\dot{a}}{a})^2 &= \frac{8\pi G_N}{3}\rho & (14) \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G_N}{3}(\rho + 3p) & (15) \end{aligned}$$

where G_N is Newton's constant. In accordance with the energy conservation equation

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} \quad = \quad 0 \tag{16}$$

and that the "trace-anomaly" energy mass den sity ρ^T_{QCD} is the preponderant part contributing to ρ, we find a nearly constant value for ρ^T_{QCD} which is, using equation (11), given by

$$\rho^T_{QCD} \quad = \quad -\frac{9\alpha_{S_0}}{32\pi}\epsilon^2 < G^2 > \tag{17}$$

We see here that the vacuum gluon condensate $< G^2 >$ should have a negative value which is not unreasonable since the inflationary vacuum has "strange" properties. In ordinary inflationary models, it is filled with repulsive-gravity matter turning gravity on its head [14]. This reversal of the vacuum properties is reflected, in our model, by a reversal of sign for the vacuum gluon condensate. Moreover, in order that the energy density stays approximately constant, we see from equation (17) that the time variation of ϵ should be very slow. This agrees with the "inflationary" solution of the FRW equations $a(t) \sim e^{Ht}$ which gives substituting (17) in (14)

$$H \equiv \frac{\dot{a}}{a} \quad = \quad \epsilon\sqrt{\frac{3\alpha_{S_0}}{4} G_N |< G^2 >|} \tag{18}$$

corresponding to a Hubble constant changing very slightly with time as required.

Hence, rewriting the equation of motion (equation 4) in the form

$$\frac{\ddot{\epsilon}}{\epsilon} + 3H\frac{\dot{\epsilon}}{\epsilon} - (\frac{\dot{\epsilon}}{\epsilon})^2 = \frac{l^2 < G^2 >}{2} \tag{19}$$

we can neglect the terms involving $\frac{\ddot{\epsilon}}{\epsilon}$ and $(\frac{\dot{\epsilon}}{\epsilon})^2$ to get

$$\epsilon(t) \quad = \quad \epsilon_i - \frac{1}{3^{3/2}(\alpha_{S_0})^{1/2}}(\frac{l}{L_p})^2 G_N^{1/2} |< G^2 >|^{1/2}(t - t_i) \tag{20}$$

where ϵ_i is the value of ϵ at t_i the initial time of inflation and we set it very close to 1 such that ϵ_f the value at the end of inflation t_f is equal, by convention, to 1.

In order that the above solution is consis tent so that the temporal change of ϵ during the inflation is very small, we should have

$$\frac{1}{3^{3/2}(\alpha_{S_0})^{1/2}}(\frac{l}{L_p})^2 G_N^{1/2} |< G^2 >|^{1/2}(t_f - t_i) << 1 \tag{21}$$

Also, neglecting the $(\frac{\dot{\epsilon}}{\epsilon})^2$ term in equation (19) amounts to the condition

$$\delta \equiv |\frac{\dot{\epsilon}}{H\epsilon}| \quad << \quad 1 \tag{22}$$

This condition is written for our solution as

$$\delta = \frac{2}{9\alpha_{S_0}}(\frac{l}{L_P})^2 \quad << \quad 1 \tag{23}$$

We should evaluate the QCD coupling constant $\alpha_{S_0}(\mu) = \frac{4\pi}{\beta_0 \ln(\frac{\mu^2}{\Lambda^2_{QCD}})}$ at an energy scale corresponding to the inflationary period. We take this to be around the GUT scale $\sim 10^{15} GeV$ and $\beta_0 = 11 - \frac{2}{3} n_f = 7$ whereas the weak logarithmic dependence would assure the same order of magnitude for α_{S_0} calcu lated at other larger scales. With $\Lambda_{QCD} \sim 0.2 GeV$ [15] we estimate $\alpha_{S_0} \sim 0.025$ and so we get the consistency condition

$$(\frac{l}{L_P})^2 \quad << \quad 0.1 \tag{24}$$

This gives us the first hint that Bekenstein hypothesis ($Ł_P < l$) might not survive in our model.

3 Results and Conclusion

Looking at equation (17) and comparing with $\rho = V(0)$ in ordinary inflationary models, we see that the gluon condensate plays a role of a potential for the "inflaton"-ϵ field. Furthermore, neglecting the $\frac{\ddot{\epsilon}}{\epsilon}$ and $(\frac{\dot{\epsilon}}{\epsilon})^2$ terms in equation (19) corresponds to "slow rolling" solutions along the potential curve

$$V'(\epsilon) \quad = \quad -\frac{l^2 < G^2 >}{2} \epsilon \tag{25}$$

Now, we check that our model is able to fix the usual problems of the standard (big bang) cosmology. Fir st, in order to solve the "horizon" and "flatness" problems we need an inflation $\frac{a(t_f)}{a(t_i)}$ of order 10^{28} implying an inflation period $\Delta t = t_f - t_i$ such that

$$H \ \ \Delta t \quad \sim \quad 65 \tag{26}$$

Furthermore, it should satisfy the constraint

$$10^{-40} s \leq \Delta t \ll 10^{-10} s \tag{27}$$

so that not to conflict with the explanation of the baryon number and not to create too large density fluctuations [17, **?**]. The bound $10^{-10} s$ corresponds to the time, after the big bang, when the electroweak symmetry breaking took place. Presumably, our inflation should have ended far before this time. Thus, from equations (26), (27) and (18) we get the following bounds on $| < G^2 > |$:

$$3 \times 10^7 GeV^2 \ll | < G^2 > |^{1/2} \leq 3 \times 10^{37} GeV^2 \tag{28}$$

Next, comes the "formation of structure" problem and we require the fractional density fluctuations at the end of inflation to be of the order $\frac{\delta M}{M} \mid_{t_f} \sim 10^{-5}$ so that quantum fluctuations in the de Sitter phase of the inflationary universe form the source of perturbations providing the seeds for galaxy formation and in order to agree with the CMB anisotropy limits. Within the relativistic theory of cosmological perturbations [19], the above fractional density fluctuations represent (to linear order) a gauge-invariant quantity and satisfy the equation

$$\frac{\delta M}{M} \mid_{t_f} \;=\; \frac{\delta M}{M} \mid_{t_i} \frac{1}{1+\frac{p}{\rho}} \mid_{t_i} \tag{29}$$

The initial fluctuations are generated quantum mechanically and, at the linearized level, the equations describing both gravitational and matter perturbations can be quantized in a consistent way [20]. The time dependence of the mass is reflected in the nontrivial form of the solutions to the mode equations and one can compute the expectation value of field operators such as the power spectrum and get the following result for the initial mass pert urbation [19, 20]

$$\frac{\delta M}{M} \mid_{t_i} \;=\; \frac{V'H}{l\rho} \tag{30}$$

whence

$$10^{-5} \sim \mid \frac{\delta M}{M} \mid_{t_f} \;=\; \mid \frac{V'H}{l} \mid_{t_i} \frac{1}{|(\rho+p)|_{t_i}} \tag{31}$$

In order to evaluate $(\rho+p)|_{t_i}$ we use the energy conservation equation (16) after substituting $\rho \sim \rho^T_{QCD}$ and we get

$$|(\rho+p)|_{t_i} \;=\; \frac{1}{24\pi}(\frac{l}{L_P})^2 | < G^2 > | \tag{32}$$

In fact, the energy conservation equation can be used to solve for ρ^r_{QCD} and we could check that

$$\rho^r_{QCD}(\dot{\rho^r_{QCD}}) \sim \rho_\epsilon(\dot{\rho}_\epsilon) \;\sim\; \delta \times \rho^T_{QCD}(\delta \times \dot{\rho^T_{QCD}}) \tag{33}$$

where $\delta \equiv |\frac{\dot{\epsilon}}{H\epsilon}| \sim (\frac{l}{L_P})^2$ and so, when the condition (24) is satisfied, our solution assuming the predominance of the "trace-anomaly" energy mass density is self-consistent. Substituting equation (32) in (31) and using eqaution (25) we get

$$\frac{l}{L_P} \sim 6\pi 10^5 \sqrt{3\alpha_{S_0}} |< G^2 >|^{\frac{1}{2}} G_N \quad (34)$$

Hence, taking $G_N \sim 10^{-38} GeV^{-2}$ we obtain

$$\frac{l}{L_P} \sim \frac{|< G^2 >|^{\frac{1}{2}}}{10^{33} GeV^2} \quad (35)$$

and combining this last result with equation (28), we get

$$10^{-26} \quad << \quad \frac{l}{L_p} \leq 10^4 \quad (36)$$

Nevertheless, the condition (21) for the small temporal change of ϵ would give, when imposing the relation (26), a bound on $(\frac{l}{L_P})^2 \sim \delta \ll \frac{1}{65}$ in conformity with the consistency condition (equation 24) but in disagreement with Bekenstein assumption that L_P is the shortest length scale in any physical theory. However, it should be noted that Beckenstein's framework is very similar to the dilatonic sector of string theory and it has been pointed out in the context of string theories[21] that there is no need for a universal relation between the Planck and the string scale. Furthermore, determining the order of magnitude of $\frac{l}{L_P}$ is interesting in the context of these theories.

To fix the ideas, let's take, say, $|< G^2 >|^{\frac{1}{2}} \sim 10^{25} GeV^2$ which would give $\frac{l}{L_P} \sim 10^{-8}$. On the other hand, we should compare the above value for $< G^2 >$ with its present value renormalized at GUT scale $\sim 10^{15} GeV$ which can be calculated knowing its value at $1 GeV$ [9] and that the anomalous dimension of $\alpha_S G^2$ is identical ly zero. We get $< G^2(now, \mu \sim 10^{15} GeV) >\sim 1 GeV^4$ which represents a decrease of 50 orders of magnitude. One, then, can imagine unification or other effects leading to the huge value at very high temperatures (whereas restoration of chiral symmetry alone might have suggested a zero value at high energy), then a phase transition of $< G^2 >$ occurs reducing its value and reversing its sign. We take the value of ϵ around which this phase transition takes place to be equal to 1 by convention. During the phase transition, the value of the condensate $< G^2 >$ drops down drastically and the energy release of this helps in reheating the universe while changing the sign would force it to pass through the value 0 leading to a minute "trace-anomaly" energy mass density (equation 17) ending, thus, the inflation. Surely, this phenomenological description needs to be tested and expanded into a theory where the concept of symmetry breaking provides the physical basis for ending the inflation. We hope this work will stimulate further interest in the subject.

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